



**Cambridge  
Elements**

Foundations of  
Contemporary Physics

**Gauge Symmetries,  
Symmetry Breaking,  
and Gauge-Invariant  
Approaches**

Philipp Berghofer,  
Jordan François,  
Simon Friederich,  
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Guy Hetzroni,  
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Elements in the Foundations of Contemporary Physics

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# Gauge Symmetries, Symmetry Breaking, and Gauge-Invariant Approaches

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**Abstract:** Gauge symmetries play a central role, both in the mathematical foundations as well as the conceptual construction of modern (particle) physics theories. However, it is yet unclear whether they form a necessary component of theories or they can be eliminated. It is also unclear whether they are merely an auxiliary tool to simplify (and possibly localize) calculations or they contain independent information. Therefore, their status, both in physics and philosophy of physics, remains to be fully clarified. This Element reviews the current state of affairs on both the philosophy and the physics side. In particular, it focuses on the circumstances in which the restriction of gauge theories to gauge-invariant information on an observable level is warranted, using the Brout–Englert–Higgs theory as an example of particular current importance. Finally, the authors determine a set of yet-to-be-answered questions to clarify the status of gauge symmetries.

**Keywords:** gauge symmetries, symmetry breaking, Higgs mechanism, dressing field method, FMS approach

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## 1 Introduction

Gauge symmetry is a central concept in essentially all of modern fundamental physics. The framework of theories in which gauge symmetries play a central role – gauge theories – is very general, and many physicists expect that any future discoveries will be accommodated within it. However, there are unresolved issues in the foundations of gauge theories, notably concerning which features of gauge theories are descriptively redundant, and which are crucial for empirical adequacy. The aim of this Element is to present precisely what is known on gauge symmetries and the possibility of gauge symmetry breaking, stressing the relevance of foundational and philosophical issues to current scientific practice and open questions, and to further outline what we take to be the most promising avenues forward. This Element is thus an invitation to anyone interested in understanding the conceptual foundations of gauge theories, and a reflection upon how these features shape the way we think about elementary fields and particles.

Most results on gauge theories stem from approaches that make some drastic simplifications. Gauge theories with weak interactions are often treated using perturbative approximations. In these approximations, many of the geometric properties of non-Abelian gauge theories, like their nontrivial topological features, play little to no role. (Lattice) simulations, suitable especially for strongly interacting theories, can be formulated in such a way that the gauge symmetry plays essentially no role in practice. Thus, the conceptual questions concerning the gauge symmetries themselves usually do not arise as problems in practice.

However, even at this point a little conceptual reflection shows that the central implicit and explicit foundational assumptions on gauge symmetries are not always consistent with one another. Gauge-dependent objects depend on the choice of gauge fixing, which is made on pragmatic grounds, not dictated by any choice of gauge made on “nature’s” behalf. This is one among several reasons why it is commonly stated that gauge-dependent objects cannot directly correspond to anything physically real. This assertion, however, casts doubt on the physical reality of elementary particles such as electrons and quarks, together with the fields that represent them. This is in sharp tension with the common discourse and with aspects of the scientific practice in which these gauge-dependent fields are taken to be physically real in the same sense as, say, atoms are usually taken to be physically real. This tension already highlights why properly understanding gauge symmetries is important from an ontological point of view.

There are three standard ways to avoid gauge-dependent objects in the treatment of the gauge interactions that form part of the Standard Model of elementary particle physics (Maas, 2019): (i) in quantum electrodynamics (QED) a so-called photon cloud dressing reestablishes gauge invariance by including, in the description of the electron, what one might characterize as its “Coulomb tail” (Haag, 1992); (ii) in quantum chromodynamics (QCD), the resolution, or rather the irrelevance of gauge dependence, is due to confinement, which requires that only uncharged (with respect to the non-Abelian color charge) and thereby gauge-invariant objects appear at distances at or beyond the radius of hadrons (Lavelle & McMullan, 1997; Yndurain, 2006); and (iii) in the electroweak sector, though much less known, it is due to the Fröhlich–Morchio–Strocchi mechanism (Fröhlich, Morchio, and Strocchi, 1980, 1981). While these three mechanisms appear quite different at first sight, they eventually all boil down to canceling gauge dependency by either eliminating the gauge degrees of freedom, or, at least, ensuring they do not appear in the empirically accessible range.

However, the fact that gauge-dependent objects can be eliminated in an unobtrusive way in the gauge theories just mentioned seems to depend on features that are specific to the theories combined in the Standard Model and may not hold in extensions of it. A more systematic strategy for eliminating gauge dependence may be necessary for future progress in the search for physics beyond the Standard Model. A “literal interpretation” of gauge fields that regards different gauge symmetry-related field configurations as physically distinct, in contrast, may well be an obstacle to such progress. Thus it is necessary to establish whether a manifestly gauge-invariant approach to gauge theories, replacing the current way of thinking about elementary particles, is compelling or perhaps even necessary for further progress. This need, as we shall see, mirrors themes in the recent philosophical discourse on gauge symmetries.

The structure of this Element is as follows. It starts out with a review of general features of (gauge) symmetries in Section 2. Many conceptual and technical complications surrounding gauge dependence arise in connection with the spontaneous breaking of gauge symmetry. The understanding of gauge symmetry breaking is particularly central in the context of the Brout–Englert–Higgs (BEH) effect, and this is discussed in Section 3. Based on this discussion, we motivate the search for gauge-invariant approaches in Section 4, and their implementation, given at various levels of detail, in Sections 5 and 6. In Section 7 we conclude with some reflections about the ultimate consequences of the results presented, and which key steps would have to be taken to answer all substantive open questions about gauge symmetries.